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III. De Fractionibus Algebraicis Radicalitate immunibus ad Fractiones Simpliciores reducendis, deque summandis Terminis quarundam Serierum æquali Intervallo a se distantibus. Auctore *Abrahamo de Moivre*, S. R. Socio.

Eruditissimo Viro *JOHANNI MACHIN*, Societatis Regalis Secretario, *A. de Moivre*, S. P.

*MITTO* tibi excerpta quædam e Chartis meis coram Regali Societate 5<sup>o</sup> Maii 1720, exhibitis, quibus eodem die manum apposuerunt Secretarii. Pars altera harum Chartarum jam per biennium apud Cl. Præsidem reposita fuerat; continebat autem Demonstrationes Propositionum quarundam in Libro a me Anglice emissò qui inscriptus est, *THE DOCTRINE OF CHANCES*. Pars altera continebat explanationem uberiores Demonstrationum quas prior levius tetigerat. Jam cum sæpius me instigasti ut selectas Propositiones quasdam ex his Chartis desumptas publici juris facerem, utpote existimans in illis quædam reperiri quæ ad res majoris momenti quam sit speculatio ludorum applicari possint; huic tuo desiderio tandem obtempero, idque eo libentius, quo mihi videor jure aliquo a Te itidem impetrare posse ut pulcherri-  
ma tua inventa in Publicum proferre diutius non relucteris. Vale.

2 August  
1722.

P R O.

## P R O P O S I T I O I.

*SI sit Fractio qualibet  $\frac{1}{1 - ex + fxx - gx^3 \&c.}$ ,  
cujus Numerator sit data Quantitas, & Denomina-  
tor sit Multinomial utcunque compositum ex datis,  
1, e, f, g, &c. & indeterminata x, dico Fractionem su-  
pradictam ad Fractiones simpliciores reducibilem fore.*

*Casus Primus.*

Sit Fractio proposita  $\frac{1}{1 - ex + fxx}$ ; finge Deno-  
minatorem  $1 - ex + fxx = 0$ , fintque  $\frac{1}{m}$ ,  $\frac{1}{p}$  ra-  
dices istius Æquationis, sive facto  $xx - ex + f = 0$ ,  
fint  $m, p$ , radices Æquationis novæ, fac  $A = \frac{m}{m - p}$ ,  
atque  $B = \frac{p}{p - m}$ , & erit Fractio proposita æqualis sum-  
mæ  $\frac{A}{1 - mx} + \frac{B}{1 - px}$ .

*Casus Secundus.*

Sit Fractio proposita  $\frac{1}{1 - ex + fxx - gx^3}$ ; fin-  
gatur  $x^3 - exx + fx - g = 0$ , fintque  $m, p, q$ , ra-  
dices istius Æquationis, pone  $A = \frac{1}{m - p \times m - q}$ ,  
 $B =$

$$B = \frac{pp}{p-m \times p-q}, C = \frac{qq}{q-m \times q-p}; \text{ \& } \\ \text{erit Fractio proposita æqualis summæ } \frac{A}{1-mx} \\ + \frac{B}{1-px} + \frac{C}{1-qx}.$$

*Casus Tertius.*

$$\text{Sit Fractio proposita } \frac{1}{1-ex+fx^2-gx^3+hx^4}, \\ \text{Fingatur } x^4-ex^3+fx^2-gx+b=0, \text{ sintque } \\ m, p, q, s, \text{ Radices istius } \text{Æquationis, pone } A = \\ \frac{m^3}{m-p \times m-q \times m-s}, B = \frac{p^3}{p-m \times p-q} \\ \times \frac{q^3}{p-s}, C = \frac{q^3}{q-m \times q-p \times q-s}, D = \\ \frac{s^3}{s-m \times s-p \times s-q}, \text{ eritque Fractio proposita } \\ \text{æqualis summæ } \frac{A}{1-mx} + \frac{B}{1-px} + \frac{C}{1-qx} + \\ \frac{D}{1-sx}.$$

*Casus Quartus.*

$$\text{Sit Fractio proposita } \frac{1}{1-ex+fx^2-gx^3+hx^4-kx^5}, \\ \text{fingatur } x^5-ex^4+fx^3-gx^2+hx-k=0, \text{ sint-} \\ \text{que } m, p, q, s, t, \text{ Radices istius } \text{æquationis; pone } A = \\ \frac{m^4}{m-p \times m-q \times m-s \times m-t}, B = \frac{p^4}{p-m \times p-q \times p-s \times p-t}, \\ C = \frac{q^4}{q-m \times q-p \times q-s \times q-t}, D = \frac{s^4}{s-m \times s-p \times s-q \times s-t}, E = \frac{t^4}{t-m \times t-p \times t-q \times t-s}, \\ \text{eritque Fractio proposita æqualis summæ } \frac{A}{1-mx} + \frac{B}{1-px} + \frac{C}{1-qx} + \frac{D}{1-sx} + \frac{E}{1-tx}.$$

$$\times \overline{p-s} \times \overline{p-t}, C = \frac{q^4}{\overline{q-m} \times \overline{q-p} \times \overline{q-s} \times \overline{q-t}},$$

$$D = \frac{s^4}{\overline{s-m} \times \overline{s-p} \times \overline{s-q} \times \overline{s-t}}, E = \frac{t^4}{\overline{t-m}}$$

$$\times \overline{t-p} \times \overline{t-q} \times \overline{t-s}. \text{ Eritque Fractio proposita}$$

$$\text{æqualis summæ, } \frac{A}{\overline{1-mx}} + \frac{B}{\overline{1-px}} + \frac{C}{\overline{1-qx}} +$$

$$\frac{D}{\overline{1-sx}} + \frac{E}{\overline{1-tx}}. \text{ Lex Reductionis ita uno intuitu se prodit, tamque facilis est illius continuatio ut in-}$$

utile foret illam verbis explanare.

### Corollarium I.

Si Radices omnes sint æquales, non poterit Fractio proposita reduci ad simpliciores.

### Corollarium II.

Si Radices aliquæ sint æquales, aliæ vero inæquales, poterit reduci fractio proposita ad simpliciores; sit v.g.

fractio proposita  $\frac{1}{\overline{1-ex} + \overline{fx} \times \overline{gx}}$ , factoque

ut præscriptum est  $x^3 - exx + fx - g = 0$ . Sint

Radices istius æquationis  $m, p, q$ , quarum  $m$  &  $p$  sint æquales: erunt fractiones simplices in quas resolvitur

$$\text{proposita } \frac{mm}{\overline{m-p} \times \overline{m-q} \times \overline{1-mx}} + \frac{p}{\overline{p-m}}$$

$$+ \frac{pp}{\overline{p-q} \times \overline{1-px}} + \frac{qq}{\overline{q-m} \times \overline{q-p} \times \overline{1-qx}};$$

addantur duæ priores in unam summam, & erit summa  
(divisis

(divisis Numeratore & Denominatore per  $m - p$ )

$$\frac{m p - q \times m + p + m p q x}{m - q \times p - q + 1 - m x \times 1 - p x}, \text{ five}$$

$$\frac{m m - 2 q m + m m q x}{m - q \times 1 - m x} \text{ five } \frac{m}{m - q \times 1 - m x}$$

$$- \frac{q m}{m - q \times 1 - m x} \text{ adeoque Fractiones reductae}$$

$$\text{erunt } \frac{m}{m - q \times 1 - m x} - \frac{q m}{m - q \times 1 - m x}$$

$$+ \frac{q q}{m - q \times 1 - q x}.$$

### Corollarium III.

Si Fractiones simplices in quas resolvitur Fractio proposita involvant Quantitates imaginarias, tunc quicquid est imaginarii semper destruetur per additionem duarum vel plurium fractionum numero pari sumptarum.

### Corollarium IV.

Ex combinatione Fractionum simplicium, & apta limitatione Radicum, plurima suborientur Theoremata in quibus inerit concinnitas quaedam minime aspernanda Ex. g. fit fractio proposita  $\frac{1}{1 - ex + fxx - gx^3 + hx^4}$  factoque ut antea  $x^4 - ex^3 + fxx - gx + h = 0$ . Sint  $m, p, q, s$ , Radices Aequationis, sintque Fractiones in quas resolvitur proposita,  $\frac{A}{1 - mx} + \frac{B}{1 - px} +$

$\frac{C}{1-qx} + \frac{D}{1-sx}$ . Ponatur  $q = -m$ , atque  
 $s = -p$ ; addantur simul duæ priores, itemque  
 duæ posteriores, & reducetur fractio proposita ad

$$\frac{m+p-mpx}{2 \times m+p \times 1-mx \times 1-px} + \frac{m+p}{m+p+mpx}$$

si vero ponatur  $p = -m$ , at-  
 que  $s = -q$ , & addantur duæ priores, itemque duæ

posteriores, reducetur Fractio proposita ad  $\frac{mm-qq}{mm-qq}$

$$\frac{mm}{1-mmx} + \frac{qq}{qq-mm \times 1-qqx}$$

## PROPOSITIO II.

*Si sit Fractio qualibet cujus Numerator sit data  
 Quantitas, & Denominator sit Trinomium vel Qua-  
 drinomialium vel Quinquinomialium, &c. radicalitate non  
 affectum & utcumque compositum ex datis, i, e, f, g, h,  
 &c. & indeterminata x atque dividator Numerator  
 per Denominatorem, ut habeatur Series Infinita;  
 dico fore ut, si sumantur Termini quilibet istius se-  
 riei equalibus intervallis a se invicem distantibus,  
 series infinitæ inde resultantes, summabiles futuræ  
 sint.*

### Exemplum I.

Sit Fractio proposita  $\frac{1}{1-x-xx}$ ; reducatur illa  
E c ad

ad seriem infinitam, nempe ad  $1 + x + 2xx + 3x^3 + 5x^4 + 8x^5 + 13x^6 + 21x^7 + 34x^8 \text{ \&c.}$  fumanturque termini omnes alterni, incipiendo a primo, itidemque fumantur Termini omnes alterni, incipiendo a secundo, hincque conficiantur series binæ,

$$\text{Videlicet, } 1 + 2xx + 5x^4 + 13x^6 + 34x^8 \text{ \&c.} \\ x + 3x^3 + 8x^5 + 21x^7 + 55x^9 \text{ \&c.}$$

Fingatur Denominator Fractionis propositæ,  $1 - x - xx = 0$ , jam cum indices potestatum indeterminatæ  $x$ , in novis seriebus se invicem superent communi differentia 2, pone  $xx = z$ , atque ope duarum æquationum  $1 - x - xx = 0$ , &  $xx = z$ , exterminetur  $x$ ; fietque  $1 - 3z + zz = 0$ ; jam nunc restituitur  $x$ , & erit  $1 - 3xx + x^4 = 0$ ; dividatur hæc æquatio per primam, quotiens erit  $1 + x - xx$ ; fumantur alternatim Termini quotientis, propter Terminos alternatim sumptos in serie proposita, hincque orientur summæ duæ,  $1 - xx$ , &  $x$ ; constituentur hæc summæ Numeratores Fractionum duarum quarum communis Denominator sit  $1 - 3xx + x^4$ . eruntque  $\frac{1 - xx}{1 - 3xx + x^4}$  &  $\frac{x}{1 - 3xx + x^4}$  summæ respectivæ novarum Serierum.

### *Exemplum II.*

Si vero desiderentur summæ terminorum intervallis binis a se distantium, fiat ut prius  $1 - x - xx = 0$ , jam cum indices potestatum in novis seriebus se invicem superent communi differentia 3, ponatur  $x^3 = z$ , & fiet  $1 - 4z - zz = 0$ , atque restituto  $x$ , fiet  $1 - 4x^3 - x^6 = 0$ ; dividatur  $1 - 4x^3 - x^6$  per  $1 - x - xx$ , quotiens erit  $1 + x + 2xx - x^3 + x^4$ , ejus termini ordinatim sumpti ad intervalla bina, tres conficient



conficient summas, videlicet,  $1 - x^3$ ,  $x + x^4$ ,  $2 x x$ , quæ figillatim sumptæ, erunt illæ Numeratores, trium Fractionum, quibus si apponatur communis Denomi-

nator  $1 - 4 x^3 - x^6$ , erunt tres Fractiones,  $\frac{1 - x^3}{1 - 4 x^3 - x^6}$ ,

$\frac{x + x^4}{1 - 4 x^3 - x^6}$ ,  $\frac{2 x x}{1 - 4 x^3 - x^6}$ , summæ tres Terminorum omnium binis intervallis a se distantium, incipiendo respective a primo, secundo & tertio Terminò ; at-

que eodem methodo colligere licet summas terminorum ternis vel quaternis vel quinis intervallis a se distantibus, siue denominator sit quadrinomial, vel multinomial quodcunque ex terminis finitis compositum.

### P R O P O S I T I O III.

*Si dividatur Unitas per Trinomial utcunque compositum ex datis  $1, e, f, g$ , &c. & indeterminata  $x$ ; dico Terminum quemvis Seriei ex hac divisione resultantis, assignabilem fore.*

Sit Trinomial  $1 - e x + f x x$  finge  $x x - e x + f = 0$ , sint  $m$  &  $p$ , radices Æquationis ; sit  $l + 1$  locus termini desiderati, hoc est exprimat  $l$  intervallum inter primum Terminum & Terminum quæsitum, fac

$A = \frac{m}{m - p}$   $B = \frac{p}{p - m}$ . Et erit Terminus deside-

tus  $\overline{A m^l + B p^l} \times x^l$ .

Eodem modo si dividatur Unitas per quadrinomial  $1 - e x + f x x - g x^3$ , pone  $x^3 - e x x + f x - g = 0$ , sintque  $m, p, q$ , radices Æquationis, fac  $A =$

$\frac{m m}{m - p \times m - q}$ ,  $B = \frac{p p}{p - m \times p - q}$ ,  $C =$

$$\frac{q q}{q - m \times q - p}. \quad \text{Et erit Terminus desideratus}$$

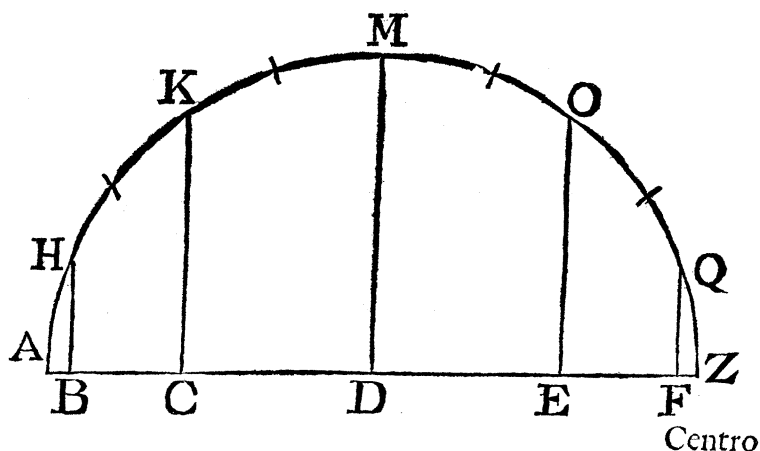
$A m' + B p' + C q' \times x'$ , & lex eadem obtinet pro multinomiis quibuscunque.

## P R O B L E M A.

*A & B quorum Dexteritates sint in ratione data videlicet ut a ad b, ea conditione ludant, ut quoties A ludum unum vicerit, B tradat ipsi nummum unum: quoties vero B vicerit, A tradat ipsi nummum unum: & non prius ludo desistant, quam eorum alter nummos omnes alterius lucratus fuerit; queritur quantum probabile futurum sit ut certamen intra datum ludorum numerum x, vel expirante illo numero, finiatur.*

### *Casus Primus.*

Sit  $n$  numerus nummorum quos uterque Collusorum habeat; sit etiam  $n$  numerus par, ponaturque  $a$  ad  $b$  habere rationem æqualitatis.



Centro D, Intervallo  $DA = 1$ , describatur Semicircumferentia  $AMZ$  quæ dividatur in tot partes æquales quot sunt unitates in  $n$ , tunc ex primo H, tertio K, quinto M &c. & impari quoque divisionis termino, demittantur ad diametrum perpendiculara  $HB, KC, MD,$

$$OE, QF \text{ \&c. ponatur } Q = \frac{HB^{x+1}}{AB^{\frac{1}{2}x+1}} - \frac{CK^{x+1}}{AC^{\frac{1}{2}x+1}} + \frac{DM^{x+1}}{AD^{\frac{1}{2}x+1}} - \frac{EO^{x+1}}{AE^{\frac{1}{2}x+1}} + \frac{QF^{x+1}}{AF^{\frac{1}{2}x+1}} \text{ \&c. donec}$$

finis omnes exhauriantur: quo facto, erit probabilitas certaminis finiendi intra ludos non plures quam  $x$ , ad probabilitatem non finiendi, ut  $2^{\frac{1}{2}x-1}n - Q$  ad  $Q$ , accurate.

### Corollarium I.

Si fumatur pro  $Q$  Terminus primus  $\frac{HB^{x+1}}{AB^{\frac{1}{2}x+1}}$

neglectis cæteris, habebitur approximatio sufficiens nisi forte sit  $x$  numerus valde exiguus.

### Exemplum.

Sit  $n$  numerus nummorum quos uterque Collusorum habeat = 10. Sit etiam  $x = 76$ . Si fumatur pro  $Q$  primus terminus & negligentur cæteri, inveniatur probabilitas certaminis finiendi intra ludos non plures quam 76 ad probabilitatem non finiendi ut 50747 ad 49235, si vero fumatur pro  $Q$  termini duo priores neglectis cæteris, inveniatur ratio probabilitatum ut 50743 ad 49247.

Coroll-

*Corollarium II.*

Invenire quotenis ludis, probabilitates certaminis finiendi & non finiendi erunt æquales.

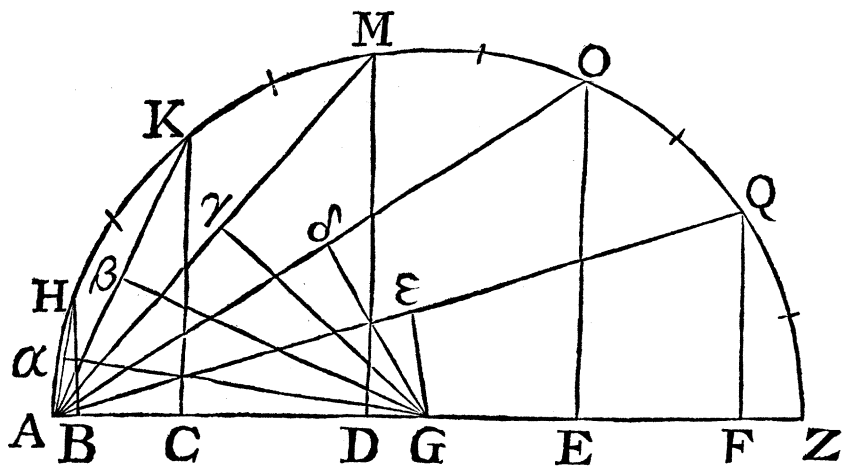
*Solutio.*

Ponatur pro Q Terminus unicus  $\frac{HB^{x+1}}{AB^{\frac{1}{2}x+1}}$ , fiatque

$2^{\frac{1}{2}x-1} n - Q = Q$ . Et posito  $n$  maximo numero invenietur,  $x = 0.756 n n$  proxime, aliquanto major quam  $\frac{3}{4}nn$ .

*Casus Secundus.*

Sit  $n$  numerus impar, ponaturque  $a$  ad  $b$  habere rationem æqualitatis.



Centro G, intervallo GA describatur semicircumferentia AMZ quæ dividatur in tot partes æquales, quot sunt

funt unitates in  $n$ ; tunc ex primo H, tertio K, quinto M, & impari quoque divisionis termino, demittantur ad diametrum perpendiculara HB, KC, MD, OE, QF &c. ex diametri extremitate A, primo scilicet arcui contermina, ducantur subtensæ AH, AK, AM, &c. ad quas e Centro G ducantur perpendiculara  $G\alpha$ ,  $G\beta$ ,

$$G\gamma, G\delta, G\varepsilon, \&c. \text{ ponatur } Q = \frac{BH^x \times G\alpha}{AB^{\frac{x+1}{2}}}$$

$$- \frac{CK^x \times G\beta}{AC^{\frac{x+1}{2}}} + \frac{DM^x \times G\gamma}{AD^{\frac{x+1}{2}}} - \frac{EO^x \times G\delta}{AE^{\frac{x+1}{2}}} +$$

$$\frac{FQ^x \times G\varepsilon}{AF^{\frac{x+1}{2}}} \&c. \text{ quo facto, erit probabilitas certaminis}$$

finiendi intra ludos non plures quam  $x$ , ad probabilitatem non finiendi, ut  $2^{\frac{x-3}{2}} n - Q$  ad  $Q$  accurate.

### Corollarium I.

Si fumatur pro  $Q$  terminus primus  $\frac{HB^x \times G\alpha}{AB^{\frac{x+1}{2}}}$  ne-

glectis cæteris, habebitur approximatio sufficiens.

### Exemplum.

Sit  $n$  numerus nummorum quos uterque Collusorum habeat = 45. Sit etiam  $x = 1519$ . Sumatur pro  $Q$  terminus primus neglectis cæteris, & invenietur probabilitas certaminis finiendi intra ludos non plures quam 1519 ad probabilitatem non finiendi ut 49959 ad 50441, quæ proportio est vero proxima.

*Corol.*

## Corollarium II.

Invenire quotenis ludis probabilitates certaminis finiendi & non finiendi erunt æquales.

*Solutio.*

Ponatur pro Q Terminus unicus  $\frac{HB^x \times G^x}{AB^{\frac{x+1}{2}}}$ , fiat.

que  $2^{\frac{x-3}{2}} n - Q = Q$ ; & posito  $n$  magno numero, inveniatur  $x = 0$ . 756  $nn$  proxime aliquanto major quam  $\frac{1}{4} nn$  contra quam sentiebat Clarissimus Monmortius.

*Casus Tertius.*

Positis cæteris ut in primo casu, sit  $a$  ad  $b$  ratio inæqualitatis (*vid. Fig. 1.*) Pone  $\frac{a^n + b^n}{a + b} = L$ ,  $\frac{a - b}{a + b} = d$ ,  $\frac{ab}{a + b} = r$ , Fac, 1, 2  $r :: \frac{HBq}{AB}$ ,  $m :: \frac{CKq}{AC}$ ,  $p :: \frac{MDq}{AD}$ ,  $q :: \frac{OEq}{AE}$ ,  $s :: \frac{QFq}{AF}$ ,  $t$ .

Pone  $Q = \frac{HB}{2rAB + d} m^{\frac{1}{2}x} - \frac{CK}{2rAC + d} p^{\frac{1}{2}x} + \frac{MD}{2rAD + d} q^{\frac{1}{2}x}$  &c. quo facto erit probabilitas ludi finiendi intra ludos non plures quam  $x$  ad probabilitatem non finiendi ut  $nr^{\frac{1}{2}n-1} = 2LQ$  ad  $2LQ$ .

*Corol.*

## Corollarium II.

Si fumatur pro  $Q$  Terminus primus  $\frac{HB}{2rAB+d}$

$m^{\frac{x}{2}}$  neglectis cæteris, habebitur approximatio sufficiens.

## Casus Quartus.

Positis cæteris ut iu secundo casu, sit  $a$  ad  $b$  ratio inæqualitatis (*vid. Fig. 2.*)

Pone quantitates  $L, d, r, m, p, q, s, t, \&c.$  ut in tertio casu.

$$\text{Pone } Q = \frac{BH \times G\alpha}{2rAB+d} m^{\frac{x-1}{2}} - \frac{CK \times G\beta}{2rAC+d}$$

$$p^{\frac{x-1}{2}} + \frac{DM \times G\gamma}{2rAD+d} q^{\frac{x-1}{2}} \&c. \text{ quo facto erit pro-}$$

babilitas ludì finiendi intra ludos non plures quam  $x$  ad probabilitatem non finiendi ut  $nr^{\frac{n-3}{2}} - 4LQ$  ad  $4LQ$ .

## Corollarium.

Si fumatur pro  $Q$  Terminus unicus  $\frac{BH \times G\alpha}{2rAB+d}$

$m^{\frac{x-1}{2}}$  neglectis cæteris habebitur approximatio sufficiens.

*Quemadmodum in Progressione Geometricâ, Terminus quilibet ad proxime præcedentem habet rationem datam, ita sunt aliæ Progressiones quæ sic constitui possunt ut assumptis ad libitum Terminis duobus primis, Terminus quilibet subsequens ad duos proxime præcedentes habeat rationes datas, hujusmodi est subiecta Series,*

F f

A B

A	B	C	D	E	F
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$$1 + 3x + 7xx + 17x^3 + 41x^4 + 99x^5, \&c. \text{ in qua}$$

$$C = 2Bx + 1Axx$$

$$D = 2 C_X + 1 B_{XX}$$

$$E = 2D_X + C_{XX}$$

$$F = 2E_x + D_{xx} \text{ \&c.}$$

*Quantitates autem Numerales 2 + 1 simul sumptas  
subque propriis signis connexas appellare licet Indi-  
cem Relationis.*

*Eodem modo constitui possunt series aliæ in quibus assumptis ad libitum Terminis tribus primis, Terminus quilibet subsequens ad tres proxime præcedentes habeat rationes datas; hujus generis est subjecta Series.*

A B C D E F

$$1 + 2x + 3xx + 10x^3 + 34x^4 + 97x^5 \&c. \text{ in qua}$$

$$D = 3Cx - 2Bxx + 5Ax^3$$

$$E = 3Dx - 2Cxx + 5Bx^3$$

$$F = 3Ex - 2Dxx + 5Cx^3 \text{ \&c.}$$

Quantitates autem Numerales  $3-2+5$  simul sumptæ subque propriis signis connexæ, componunt Indicem Relationis.

*Sunt aliæ series in quibus Relatio fit ad quatuor, vel ad quinque, vel ad sex Terminos præcedentes, &c.*

Series autem omnes hujus generis recurrentes appellare licebit propter Relationem Terminorum perpetuo recurrentem.

## PROBLEMA II.

*In seriebus recurrentibus, ex datis Terminis duobus primis, si relatio fiat ad duos præcedentes; vel datis*



*dati Terminis tribus primis, si relatio fiat ad tres præcedentes, &c. dato etiam indice relationis, invenire summam Terminorum quotlibet quorum numerus datus sit.*

Problema solvitur in Tractatu nostro qui inscribitur, *The Doctrine of Chances.*

### P R O B L E M A III.

*Assumptis ad libitum seriebus quocunque recurrentibus; Terminisque, iisdem intervallis a principio serierum distantibus, in se invicem multiplicatis, invenire summam seriei ex hac multiplicatione resultantis.*

### I N V E S T I G A T I O.

I<sup>o</sup> Proponantur series duæ, sitque  $m \div n$  Index Relationis in prima serie, atque  $p \div q$  Index Relationis in secunda, ex primo Indice  $m \div n$ , formetur Æquatio  $x x - m x - n = 0$ , ex secundo Indice  $p \div q$ , formetur Æquatio  $y y - p y - q = 0$ , pone  $x y = z$ . Atque ope trium istarum Æquationum, expungantur  $x$  &  $y$ , & orietur Æquatio  $z^4 - m p z^3 - m m q z z - m n p q z + n n q q = 0$ .

$$\begin{array}{r} - m p z^3 \\ - m m q z z \\ - m n p q z \\ + n n q q \end{array}$$

in qua deleto primo termino  $z^4$ , mutatis signis omnibus, atque posito  $z = 1$ , obtinebitur Index Relationis, quo obtento, series resultans facile summabitur; II<sup>o</sup> eodem modo procedere licet, si dentur series tres vel quatuor &c. recurrentes.

*Dum superiores paginae praelo subiciebantur, incidi fortuito in Acta Leipf. annorum 1702 & 1703, quibus comperi Cla. Leibnitium eadem fere methodo ante me usum fuisse qua hic utor in reducendis Fractionibus Algebraicis ad simpliciores, quod monitum velim ut a me avertam vel minimam suspicionem, aliena mihimet arrogare voluisse; Propositio autem qua id praestitimus aequae ac Propositio nostra tertia, ambae deducuntur tanquam Corollaria ex altera Propositione maxime generali quam exhibuimus coram Regali Societate, Maii 5° 1720; Propositio sic se habet.*

*Data serie quavis recurrente quarum Termini quotlibet primi ad libitum sumantur; dato etiam Indice Relationis Terminorum sequentium ad praecedentes, invenire Terminum quemlibet assignatum in hac serie, priusquam summa seriei sit cognita.*

*Qui autem rite perspexerit Solutionem Problematis hic adducti, is utique percipiet illam pendere a Propositione nostra generali, cujus demonstrationem simulque modum investigationis brevi spero publici juris faciam.*